Elements
of
Matrix Modeling and Computing
with
MATLAB

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Preface

An important objective of this book is to provide "math-on-time" for second year students of science and engineering. The student should have had one semester of calculus. The student most likely would take this matrix course concurrently with the second semester of calculus or would use this text for independent study of these important topics. This text fills in often missed topics in the first year of calculus including complex numbers and functions, matrices, algebraic systems, curve fitting, elements of linear differential equations, transform methods and some computation tools.

Chapters one and two have introductory material on complex numbers, 2D and 3D vectors and their products, which are often covered in the beginning of multivariable calculus. Here a connection is established between the geometric and algebraic approaches to these topics. This is continued into chapters three, four and five where higher order algebraic systems are solved via row operations, inverse matrices and LU factorizations. Linearly independent vectors and subspaces are used to solve over and under determined systems. Chapters six and seven describe first and second order linear differential equations and introduce eigenvalues and eigenvectors for the solution of linear systems of initial value problems. The last two chapters use transform methods to filter distorted images or signals. The discrete Fourier transform is introduced via the continuous versions of the Laplace and Fourier transforms. The discrete Fourier transform properties are derived from the Fourier matrix representation and are used to do image filtering in the frequency domain.

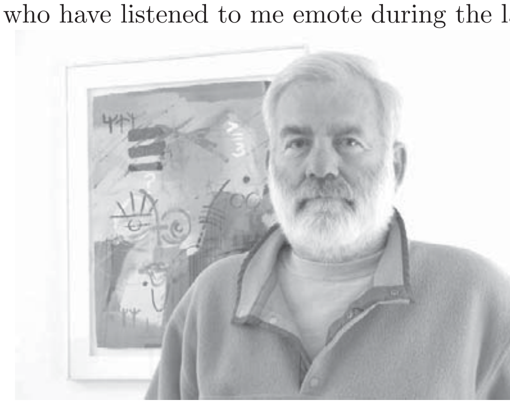
The first five chapters can be used as a two-credit course (28 50-minute classes). Among the nine chapters there is more than enough material for a three-credit course. This three-credit matrix course when coupled with a nine or ten-credit calculus sequence can serve as a more "diverse" alternative to the traditional twelve-credit calculus sequence. The twelve-credit calculus sequence can be adapted to this alternative by reducing the precalculus, moving some of 2D and 3D vectors and differential equations into the matrix course, and using computing tools to do the complicated computations and graphing.

Most sections have some applications, which should indicate the utility of the mathematics being studied. Seven basic applications are developed in various sections of the text and include circuits, trusses, mixing tanks, heat conduction, data modeling, motion of a mass and image filters. The applications are developed from very simple models to more complex models. The reader can locate sections pretaining to a particular application by using the index.

MATLAB® is used to do some of the more complicated computations. Although the primary focus is to develop by-hand calculation skills, most sections at the end have some MATLAB calculations. The MATLAB m-files used in the text are listed in the index and are included in the book's www site: http://www4.ncsu.edu/~white. The approach to using computing tools includes: first, learn the math and by-hand calculations; second, use a computing tool to confirm the by-hand calculations; third, use the computing tool to do more complicated calculations and applications.

I hope this book will precipitate discussions concerning the core mathematical course work that science and engineers are required to study. Discrete models and computing have become more common, and this has increased the need for additional study of matrix computation, numerical and linear algebra. The precise topics, skills, theory and appropriate times to teach these is certainly open for discussion. The matrix algebra topics in this book are a small subset of most upper level linear algebra courses, which should be enhanced and taken by a number of students. This book attempts to make a bridge from two and three variable problems to more realistic problems with more variables, but it emphasizes skills more than theory.

I thank my colleagues who have contributed to many discussions about the content of this text. And, many thanks goes to my personal friends and Liz White who have listened to me emote during the last year.



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Introduction

One can view an $m \times n$ matrix as a table of objects with m rows and n columns. The objects are usually real or complex numbers, but they could be characters or records of information. A simple example is data for the last 12 months of car sales where there are $m = 12$ rows and $n = 2$ columns. The first column will have the month's number and the second column will have the number of cars sold in the corresponding month. By examining the data one would like to make a prediction about futures sales. This is where the modeling enters. If the graph of the sales versus months "looks" like a straight line, then the data may be modeled by a linear function of time $y = \widehat{m}t + c$. The slope $\widehat{m}$ and intercept c must be chosen so that the computed sales are "close" to the car sales data. This is done by appropriate manipulations of the two column vectors and computing a solution of the resulting system of algebraic equations. Once $\widehat{m}$ and c have been found, the predicted sales for t larger than 12 can easily be calculated by evaluating the linear function. The modeling process is complicated by incorrect sales data, changing prices and other models such as a parabolic function of time.

This text examines a variety of applications, which have matrix models and often have algebraic systems that must be solved either by-hand calculations or using a computing tool. Applications to projectiles, circuits, mixing tanks, trusses, heat conduction, motion of a mass, curve fitting and image enhancement will be initially modeled in very simple ways and then revisited so as to make the model more accurate. This is typical of the modeling process where there is an application, a model, mathematical method, computations and assessment of the results. Then this cycle is repeated so as to enhance the application's model.

The first two chapters deal with problems in two and three dimensional space where the matrices have no more than three rows or columns. Here geometric insight can be used to understand the models. In Section 2.5 the extension to higher dimensions is indicated for vectors and matrices, solution to larger algebraic systems, more complicated curve fitting, time dependent problems with systems of differential equations and image modeling. Chapters three, four and five have the basic matrix methods that are required to solve systems in higher dimensions. Chapters six and seven contain time dependent models and introduce linear systems of differential equations. The last two chapters are an introduction to image and signal processing.

Most sections have some by-hand matrix calculations in the numbered examples, some applications and some MATLAB computations, see [4] and [6]. The focus is on the by-hand calculations, and one should carefully study the numbered examples. Each numbered example usually has two exercises associated with it. There are also additional exercises, which may fill in some parts of the text, be related to applications or use MATLAB. This text is not intended to be a tutorial on MATLAB, but there are a number of short codes that may help you understand the topics being discussed. The by-hand calculations should be done, and MATLAB should be used to confirm these calculations. This will give you confidence in both your understanding of the by-hand matrix computation and the use of MATLAB. Larger dimensional problems can easily be done using MATLAB or other computer software.

The following matrices are used in Chapters 3, 4, 5 and 9, and they can be generalized to larger matrices enabling one to cross the bridge from models with few variables to many variables.

$$Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{32}(-3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 7 & -10 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

$$[A \;\; d] = \begin{bmatrix} 2 & -1 & 0 & 200 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & 70 \end{bmatrix} \qquad [U \;\; \widehat{d}] = \begin{bmatrix} 1 & -1 & 0 & 200 \\ 0 & 3/2 & -1 & 100 \\ 0 & 0 & 4/3 & 410/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \qquad A^{-1} = (1/10) \begin{bmatrix} 8 & 6 & 4 & 2 \\ 6 & 12 & 8 & 4 \\ 4 & 8 & 12 & 6 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$$LS = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} \qquad REF = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbb{F}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & z & z^2 & z^3 \\ 1 & z^2 & 1 & z^2 \\ 1 & z^3 & z^2 & z \end{bmatrix}$$